Experiment of Magnetic Resonant Coupling Three-phase Wireless Power Transfer

Yusuke Tanikawa, Masaki Kato, Takehiro Imura, Yoichi Hori
What is our research?
0. What is our study? : Movie

1. Background of Wireless Power Transfer
   1.1. General information of WPT
   1.2. Why three phase?

2. Approach of our study
   2.1. General info. & parameters
   2.2. Theoretical formula
   2.3. Experiment and result

3. Conclusion and future works
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Wireless Power Transfer (WPT)

Applications: **EV**, mobile, and battery charger

Conventional WPT: **Single phase AC** transfer
### Three methods of WPT technologies

<table>
<thead>
<tr>
<th>Methods</th>
<th>Magnetic Induction</th>
<th>Magnetic Resonant Coupling</th>
<th>Microwave</th>
</tr>
</thead>
<tbody>
<tr>
<td>Images</td>
<td>![Image 1]</td>
<td>![Image 2]</td>
<td>![Image 3]</td>
</tr>
<tr>
<td>Transfer Gap</td>
<td>Up to 0.2 m</td>
<td>Up to 10 m</td>
<td>A few hundred km</td>
</tr>
<tr>
<td>Efficiency</td>
<td>Up to 95%</td>
<td>Up to 97%</td>
<td>Up to 60%</td>
</tr>
<tr>
<td>Robustness for Displacement</td>
<td>Poor</td>
<td>Great</td>
<td>Good</td>
</tr>
</tbody>
</table>
1.1. General info. for WPT 3/3

Magnetic Resonant Coupling

- Transmitter and receiver: LC resonators
- High efficiency (around 90%) at 1 m gap
- Transfer frequency: 100 kHz to 20 MHz
  - Power devices for kHz switching
- Relay resonators (single phase)
  - Suitable for charging EVs while running
1.2. Why three phase? 1/3

1. Advantage for **high power transfer**

2. Suitable characteristics for WPT
1.2. Why three phase? 2/3

Advantages for high power transfer

- Larger power capacity with same devices
  - Capacity limit: three times of single phase
    (High frequency devices: Low power limit)
- Relatively simple system

<table>
<thead>
<tr>
<th></th>
<th>Single phase</th>
<th>Three phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power limit ratio</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>Device quantity</td>
<td>2 (4 switches, 2 lines)</td>
<td>3 (6 switches, 3 lines)</td>
</tr>
</tbody>
</table>

Circuits
1.2. Why three phase? 3/3

Suitable characteristics for WPT

- Smaller ripples of rectified DC

- Rotation symmetry: contact-less terminal

Conventional method
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3. Conclusion and future works
1. Comparison of data and theoretical formula
2. Rotation experiment: collection of measured values
   - Each 15 degree from 0 to 360 degree
   - Mutual inductance between resonators
   - Ratio of voltage (V), current (I), and power (P)
2.1. General info. & parameters 2/2

Voltage ratio

\[ A_v = \frac{V_2}{V_1} \]

Current ratio

\[ A_i = \frac{I_2}{I_1} \]

Coupling coefficient (nondimensionalized mutual inductance)

\[ k = \frac{L_m}{L_1} = \frac{L_m}{L_2} = \frac{L_m}{880 \ \mu H} \]

Efficiency (Power ratio)

\[ \eta = \frac{P_{out}}{P_{in}} = \frac{V_2 \cdot I_2}{V_1 \cdot I_1} = A_v \cdot A_i \]

Power factor = 1.0 (resonant condition)
2.2. Theoretical formula 1/3

Base theoretical formula of single phase

Resonance: simple formula of mutual inductance and load resistance

\[ \omega_0 = \sqrt{L_1C_1} = \sqrt{L_2C_2} \]

At experiment; \( R_L = 50 \text{ ohm} \)

- **Voltage ratio**
  \[ A_v = \frac{V_2}{V_1} = j \frac{\omega_0 L_m R_L}{R_1 R_L + R_1 R_2 + (\omega_0 L_m)^2} = A_v(L_m, R_L) \]

- **Current ratio**
  \[ A_i = \frac{I_2}{I_1} = j \frac{\omega_0 L_m}{(R_L + R_2)} = A_i(L_m, R_L) \]

- **Efficiency (Power ratio)**
  \[ \eta = A_v \cdot A_i = \frac{(\omega_0 L_m)^2 R_L}{(R_L + R_2)\{R_1 R_L + R_1 R_2 + (\omega_0 L_m)^2\}} \]

2.2. Theoretical formula 2/3

Modification of formula to three phase

• Three phase; a lot of mutual inductance values
  To adapt the single phase formula;
  how to combine the values into one value?

  • Combination of trigonometric function

\[
\tilde{L}_m = L_{m_1} \sin(\omega t) + L_{m_2} \sin(\omega t + \frac{\pi}{3}) + L_{m_3} \sin(\omega t + \frac{2\pi}{3})
\]

\[
= \sqrt{(L_{m_1}^2 + L_{m_2}^2 + L_{m_3}^2) - (L_{m_1}L_{m_2} + L_{m_2}L_{m_3} + L_{m_3}L_{m_1})} \sin(\omega t + \phi)
\]

Combined \( L_m \)

\[
\sin \phi = \frac{(\sqrt{3}/2) L_{m_2} - (\sqrt{3}/2) L_{m_3}}{\sqrt{(L_{m_1}^2 + L_{m_2}^2 + L_{m_3}^2) - (L_{m_1}L_{m_2} + L_{m_2}L_{m_3} + L_{m_3}L_{m_1})}}
\]

\[
\cos \phi = \frac{L_{m_1} - (L_{m_2}/2) - (L_{m_3}/2)}{\sqrt{(L_{m_1}^2 + L_{m_2}^2 + L_{m_3}^2) - (L_{m_1}L_{m_2} + L_{m_2}L_{m_3} + L_{m_3}L_{m_1})}}
\]
2.2. Theoretical formula 3/3

Optimal load for efficient maximization

- Efficient formula: function of load resistance

\[
\eta = A_v \cdot A_i = \frac{\omega_0 (L_m R_L)^2}{(R_L + R_2)\{R_1 R_L + R_1 R_2 + (\omega_0 L_m)^2\}}
\]

- Optimal value of load resistance

\[
R_L = \sqrt{R_2^2 + \frac{R_2}{R_1} (\omega_0 L_m)^2}
\]
2.3. Experiment 1/7 (Setting)

- Resonant frequency: 120 kHz
- Cable: Y connection
  - Rotation symmetry
  - 50 ohm load resistance

### Resonators

<table>
<thead>
<tr>
<th></th>
<th>Phase a</th>
<th>Phase b</th>
<th>Phase c</th>
</tr>
</thead>
<tbody>
<tr>
<td>Self inductance [μH]</td>
<td>$8.73 \times 10^2$</td>
<td>$8.68 \times 10^2$</td>
<td>$8.69 \times 10^2$</td>
</tr>
<tr>
<td>Internal resistance [Ω]</td>
<td>1.21</td>
<td>1.23</td>
<td>1.12</td>
</tr>
<tr>
<td>Resonant capacitance [pF]</td>
<td>$2.04 \times 10^3$</td>
<td>$2.09 \times 10^3$</td>
<td>$2.04 \times 10^3$</td>
</tr>
<tr>
<td>Quality factor</td>
<td>$5.46 \times 10^2$</td>
<td>$5.36 \times 10^2$</td>
<td>$5.89 \times 10^2$</td>
</tr>
<tr>
<td>Resonant frequency [kHz]</td>
<td>$1.19 \times 10^2$</td>
<td>$1.19 \times 10^2$</td>
<td>$1.20 \times 10^2$</td>
</tr>
</tbody>
</table>

**Cable:** 2 mm²  
**Diameter:** 1.7 mm  
**Pitch of coil:** 2.5 mm  
**Capacitor:** 2000 pF
2.3. Experiment 2/7 (Result 1/6)

Mutual inductance

(Coupling coefficient $k$)

Combined mutual inductance; calculated from measured values

$$k = \frac{L_m}{L_1} = \frac{L_m}{L_2} = \frac{L_m}{880 \, \mu H}$$

Close position: small rotation angle
Strong coupling
2.3. Experiment 3/7 (Result 2/6)

Optimal load resistance $Z_L$

$$R_L = \sqrt{R_2^2 + \frac{R_2^2}{R_1} (\omega_0 L_m)^2}$$

Similar shape to mutual inductance
2.3. Experiment 4/7 (Result 3/6)

Phase difference $\phi$ of voltage

- 0-45 degree; phase = -90 degree
- 60 degree; phase = 180 degree
- 75-120 degree; phase = 150 degree

Precise shape
Between calculation and measured value

$$\sin \phi = \frac{(\sqrt{3}/2)A_2 - (\sqrt{3}/2)L_3}{\sqrt{(L_1^2 + L_2^2 + L_3^2) - (L_1L_2 + L_2L_3 + L_3L_1)}}$$

$$\cos \phi = \frac{L_1 - (L_2/2) - (L_3/2)}{\sqrt{(L_1^2 + L_2^2 + L_3^2) - (L_1L_2 + L_2L_3 + L_3L_1)}}$$
Voltage ratio $A_v$

\[ A_v = \frac{j \omega_0 L_m R_L}{R_1 R_L + R_1 R_2 + (\omega_0 L_m)^2} \]

- At optimal load; circular shape
- At 60 degree, Voltage ratio is maximum.
- Calculated value $< $ Measured value
2.3. Experiment 6/7 (Result 5/6)

Current ratio $A_i$

At optimal load; circular shape

At 60 degree, Current ratio is minimum.

Calculated value > Measured value

$A_i = j \frac{\omega_0 L_m}{(R_L + R_2)}$
2.3. Experiment 7/7 (Result 6/6)

Efficiency (Power ratio)

- $R_L=50\,\Omega$ (measured)
- $R_L=50\,\Omega$ (calculated)
- $R_L=$ optimal (calculated)

- 0-45 degree: 80%
- 60 degree: 35%
- 75-120 degree: 80%

Maximum deference of calculated and measured value; at 60 degree
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   2.3. **Experiment** and result

3. Conclusion and future works
3. Conclusion & future works

• Experiment and analysis of Three-phase WPT
  – 1. Comparison of data and theoretical formula
    Each formula draws the similar calculated value as measured value
  – 2. Rotation experiment
    Rotation symmetry of transmitters are shown in experimental result

• Future works
  – High power experiment and analysis
  – Coupling analysis as cross coupling.
Advanced experiments are undergoing...
Appendix
Transmitter voltage

Receiver voltage

DC-DC Efficiency

Power

Voltage

Current

Device efficiency

WPT efficiency

\[ \eta = \frac{0.737}{0.884} = 0.830 = 83\% \]
回路構成

DC電源
可変電圧
定電流モード有り

高周波インバータ

共振器

整流器

負荷
50Hzインバータ+Leaf
または
20Ω琺瑯抵抗

50Hz Inv.

Load (Leaf)
\[ \begin{align*}
A_1 \sin(\omega t) + A_2 \sin(\omega t + \frac{\pi}{3}) + A_3 \sin(\omega t + \frac{2\pi}{3}) \\
= A_1 \sin(\omega t) \\
+ A_2 \left\{\sin(\omega t) \cos \frac{\pi}{3} + \cos(\omega t) \sin \frac{\pi}{3}\right\} \\
+ A_3 \left\{\sin(\omega t) \cos \frac{2\pi}{3} + \cos(\omega t) \sin \frac{2\pi}{3}\right\} \\
= \left(A_1 - \frac{A_2}{2} - \frac{A_3}{2}\right) \sin(\omega t) + \left(\frac{\sqrt{3}}{2} A_2 - \frac{\sqrt{3}}{2} A_3\right) \cos(\omega t) \\
= \sqrt{\left(A_1 - \frac{A_2}{2} - \frac{A_3}{2}\right)^2 + \left(\frac{\sqrt{3}}{2} A_2 - \frac{\sqrt{3}}{2} A_3\right)^2} \sin(\omega t + \phi) \\
A_1 \sin(\omega t) + A_2 \sin(\omega t + \frac{\pi}{3}) + A_3 \sin(\omega t + \frac{2\pi}{3})
\end{align*} \]
対称Y形起電力において
\[ e_a = E_{m1} \sin(\omega t - 0^\circ) + E_{m3} \sin 3\omega t + E_{m5} \sin(5\omega t - 0^\circ) + \ldots \]
\[ e_b = E_{m1} \sin(\omega t - 120^\circ) + E_{m3} \sin 3\omega t + E_{m5} \sin(5\omega t - 240^\circ) + \ldots \]
\[ e_c = E_{m1} \sin(\omega t - 240^\circ) + E_{m3} \sin 3\omega t + E_{m5} \sin(5\omega t - 120^\circ) + \ldots \]
各線間電圧は
\[ e_{ab} = e_a - e_b = E_{m1}\{\sin \omega t - \sin(\omega t - 120^\circ)\} \]
\[ + E_{m5}\{\sin 5\omega t - \sin(5\omega t - 240^\circ)\} + \ldots \]
線間電圧に三倍次高調波はあらわれない
Y形結線における高調波

第1次調波など（正相）

第3次調波など（同相）

第5次調波など（逆相）

\[ e_a + e_b + e_c = 0 \]

\[ e_a + e_b + e_c = 3E_m3 \sin 3\omega t \]

\[ e_a + e_b + e_c = 0 \]

平衡回路でも、同相の高調波があれば中性点間に電流が流れる
第1次調波など（正相） 第3次調波など（同相） 第5次調波など（逆相）

\[ e_a + e_b + e_c = 0 \]

\[ e_a + e_b + e_c = 3E_m \sin 3\omega t \]

\[ e_a + e_b + e_c = 0 \]

平衡回路でも同相の高調波があれば起電力回路に循環電流が流れる
0 phase current

\[ e_a = e(t) = E_m \sin \omega t + E_m \sin 3\omega t + E_m \sin 5\omega t + \ldots \]

\[ e_b = e(t - T/3) = E_m \sin \omega (t - T/3) + E_m \sin 3\omega (t - T/3) + E_m \sin 5\omega (t - T/3) + \ldots \]

\[ = E_m \sin (\omega t - 120^\circ) + E_m \sin (3\omega t) + E_m \sin (5\omega t - 240^\circ) + \ldots \]

\[ e_c = e(t - 2T/3) = E_m \sin (\omega t - 240^\circ) + E_m \sin (3\omega t) + E_m \sin (5\omega t - 120^\circ) + \ldots \]
Previous research

• induction
Analysis for previous research

• Induction without magnetic resonant coupling

Ladder coil transmitter for rail transport
  – Small air gap
  – Three phase receivers are original of us