Experiment of Magnetic Resonant Coupling

Three-phase Wireless Power Transfer

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Abstract

Three-phase electric power (three-phase) is an essential technology for high power electric wiring. Three-phase, however, has not been attached to wireless power transfer (WPT). The authors suggest that characteristics of three-phase system are suitable for high power WPT with magnetic resonant coupling. Three-phase WPT has three times as large capacity as conventional single-phase WPT has. The power capacity of three-phase system is expected up to 10 kW or more in large air gap with high transfer efficiency. Three-phase system has two other advantages, relatively simpler circuit, and very small power ripples compared to single-phase system. These are potential advantage of three-phase system. Some suitable applications, on the other hand, are expected for three-phase system. One example of suitable application for three-phase WPT is power transfer to rotating equipment, because three-phase has rotation symmetry, thus the authors construct rotating resonators. The experiment, at first, proves feasibility of three-phase WPT. Then the rotating experiment proves three-phase WPT can be attached to rotating equipments. These results of experiments are compared to theoretical values which are calculated by equivalent circuit formulas. The formulas are modified from single-phase WPT equivalent circuit. The formulas can also derive optimal load resistance value for the highest transfer efficiency. The formulas suggest that the optimal resistance value depends on the rotating angle of resonators.

Keywords: wireless power transfer, magnetic resonant coupling, three-phase electricity, equivalent circuit

1 Introduction

Magnetic resonant coupling [1, 2] is a method for wireless power transfer (WPT) using LC resonance between transmitter and receiver. Both transmitter and receiver are called resonators, which have series connection of coil and capacitor, and have the same resonant frequency. Conventional WPT is single phase and the power that can be transferred by single phase system is low up to 3 kW. The author introduces a novel system, three-phase WPT [3]. Three-phase WPT via magnetic resonant coupling has advantages and interesting characteristics. In this paper, the authors show the advantage, feasibility, and characteristic analysis of three-phase WPT in 6 sections. The first section is here and the second shows why the characteristic of three-phase is suitable for magnetic resonant coupling. The third section shows the experiment and result, which includes examination of rotating resonators. The fourth section shows the calculation method with
equivalent circuit, which includes theoretical value of the experiment and load resistance value calculation for the highest transfer efficiency. The fifth section is discussion and the final section is conclusion.

2 Three-phase WPT’s advantage
Three-phase WPT with magnetic resonant coupling has three advantages as follows; high power capacity up to 10 kW order, the relatively simple circuit system compared to the same capacity single phase systems, and capability of continuous power transfer without power ripples.

2.1 High power capacity of WPT via magnetic resonant coupling
First of all, three-phase is effective for increasing capacity of magnetic resonant coupling WPT up to 10 kW order. Although 10 kW is already realized in magnetic induction [4, 5], the air gap between transmitter and receiver is no more than 0.1 m, and the transfer efficiency is low. The magnetic resonant coupling has advantage of transfer efficiency in larger air gap. For instance, 90 % transfer efficiency at 0.5m air gap is realizable. Even though magnetic resonant coupling has this advantage of efficiency and larger air gap, transfer power capacity is smaller than induction method. The power capacity of resonant system is limited by dielectric voltage of devices and higher transfer frequency. Dielectric voltage of capacitor on resonator is the most important limiting factor. Magnetic resonant system uses resonators, which include inductor and the capacitor, and the resonators have high quality factor $Q$. When the resonators are in resonance mode, voltage between the capacitor terminals is $Q$ times as high as transfer voltage. This means the terminal voltage is more than 6000V when the transfer voltage is 30V and $Q$ is 200. This high terminal voltage with huge current often damages the devices, so the high power transfer is difficult. On three-phase system, on the other hand, the terminal voltage is $1/\sqrt{3}$ times as high voltage as the single phase system, when the transfer power is the same. This means the dielectric voltage get a safety margin. This margin, in other words, is capacity for enlarging transfer power. The other limit of power is frequency. Magnetic resonant coupling uses 100 kHz and higher frequency compared to induction, which uses 10 to 20 kHz. Higher frequency is inevitable to maintain high efficiency with large air gap. The frequency however, is very tough for high-power semiconductor devices, therefore special fast-speed semiconductor power devices, such as SiC MOS-FET, are necessary. These fast semi-conductors have power limit up to a few kW, therefore the conventional single-phase wireless power transfer also has a limit of capacity. Three-phase, on the other hand, has three times higher power capacity using the same devices as single-phase circuit. The author, therefore, consider that three phase transfer should be a breakthrough for high power wireless transfer up to 10 kW order, which is suitable capacity for electric vehicle and other transportation, or industrial devices.

2.2 Relatively simple circuit system for high power WPT
The previous subsection shows that three-phase WPT system with magnetic resonant coupling has a potential for three times as high power capacity as the single phase system has. This higher power capacity may be possible with parallel connection of three single-phase systems. Three-phase system, however, is simpler than parallel connection of single-phase systems. As figure 1 shows, three-phase system requires three resonators for each transmitter and receiver, which is three times as many as single-phase, but the connecting wire between resonators are only three lines and switching devices require six semiconductor devices. The parallel connection of three single-phases, on the other hand, require six lines and twelve devices, which means each single-phase system requires two lines and four devices. This means three-phase system is able to reduce the number of lines and devices even maintaining transfer power. Reduction of number of semiconductor devices is also necessary to make an economical high power WPT system because the fast semiconductor, which is necessary to maintain high frequency, is expensive.
2.3 Continuous power transfer without power ripples

Continuous power transfer is also an important characteristic of three-phase. This characteristic is more reasonable for WPT. WPT uses high frequency AC, so the transferred electricity has to be extracted as DC through rectifier. The rectified DC has ripples, which is not suitable for other equipments, which is driven by the transferred electricity. The DC, rectified from single-phase AC, has full-magnitude voltage ripple, which means the ripple voltage is between 0 to peak of AC voltage. The DC from three-phase, in contrast, has only 17% magnitude of ripple voltage compared to single-phase.

3 Experiment of three-phase WPT with rotating resonators

The previous sections show advantages of three-phase WPT. This section shows an application of three-phase system through experiment. One of the suitable applications for three-phase system is rotating system, such as turntable on construction machinery between crawler and body. This application requires high power transfer, so three-phase is suitable for this application. Three-phase system, moreover, can be built as rotation symmetry. The rotation symmetry system should be suitable for rotating transfer system, thus the authors ourselves construct the symmetrical transfer system as shown in figure 1. This system has three resonators for transmitter, three resonators for receiver and each of the three resonator coils forms two equilateral triangles; transmitter triangle and receiver triangle. Both triangles have a common rotating axis, which crosses both triangles’ gravity centers and vertical to both triangle surfaces. Although this experiment is aiming for high power up to 10 kW, this experiment is conducted on a few watt of electricity because Japanese law of radio wave regulates the limit of power up to 50W, which is the limit without certification from government office.

3.1 Purpose of the experiment

Purpose of experiment is three points as follow; proof of feasibility of three-phase WPT system, to obtain the characteristics of rotating resonators in each rotating angle and, to evaluate calculation method of the theoretical value. Calculation method is explained in the next section, “equivalent circuit calculation”.

3.2 Design of experimental equipment

The equipment should be built without any characteristic for magnetic field, so all receiver holders are made of wood. The transmitter, which includes 3 resonators, is put on a turn table, which is made of styrene foam. Setting of the equipment is shown on figure 2. Power source is a multi-channel function generator.

3.3 Configuration of experiment

The resonators’ parameters are shown on table 1. 3 transmitters (or receivers) shape 0.5 m equilateral triangle on a side. The air gap between transmitter and receiver is 0.15m. Load resistance $R_l$ is 50 ohm for each phase. The circuit setting is shown on figure 3. Wire connection is Y connection to each resonator. Measurement parameters are as follow; mutual inductance $L_m$ between each resonator, input voltage $V_i$, input current $I_i$, output voltage $V_o$, output current $I_o$, and phase angle $\phi$. These parameters are source of the calculated value as follow; coefficient of coupling $k$, which is calculated with self inductance of resonators $L_1$ and $L_2$, voltage ratio $A_v$, current ratio $A_i$, and transfer efficiency $\eta$. Rotation of Transmitter is from $0^0$ to $360^0$, i.e.1 turn. Measurement is taken every $15\pm1.5^0$ of rotation; $0^0$, $15^0$, $30^0$, $45^0$, to $360^0$. Configuration of parameters and symbol characters are shown in table 2.

$$k = \frac{I_o}{\sqrt{L_1 L_2}} = \frac{I_m}{870[\mu H]}$$  \hspace{1cm} (1)

$$A_v = \frac{V_2}{V_1}$$  \hspace{1cm} (2)

$$A_i = \frac{I_2}{I_1}$$  \hspace{1cm} (3)

$$\eta = \frac{V_2 I_2}{V_1 I_1} = A_v \cdot A_i$$  \hspace{1cm} (4)
### Table 1: Parameters of resonators

<table>
<thead>
<tr>
<th>Phase of Resonator</th>
<th>Phase a (μH)</th>
<th>Phase b (μH)</th>
<th>Phase c (μH)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transmitter</td>
<td>8.73×10¹</td>
<td>8.68×10¹</td>
<td>8.69×10¹</td>
</tr>
<tr>
<td>Self inductance</td>
<td>8.67×10²</td>
<td>8.69×10²</td>
<td>8.69×10²</td>
</tr>
<tr>
<td>Internal resistance</td>
<td>1.21</td>
<td>1.23</td>
<td>1.12</td>
</tr>
<tr>
<td>Capacitance (pF)</td>
<td>2.04×10⁴</td>
<td>2.09×10⁴</td>
<td>2.04×10⁴</td>
</tr>
<tr>
<td>Q</td>
<td>5.46×10⁵</td>
<td>5.36×10⁵</td>
<td>5.89×10⁵</td>
</tr>
<tr>
<td>Resonant frequency (kHz)</td>
<td>1.19×10⁶</td>
<td>1.19×10⁶</td>
<td>1.20×10⁶</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Phase of Resonator</th>
<th>Phase a (μH)</th>
<th>Phase b (μH)</th>
<th>Phase c (μH)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Receiver</td>
<td>8.73×10¹</td>
<td>8.69×10¹</td>
<td>8.69×10¹</td>
</tr>
<tr>
<td>Self inductance</td>
<td>8.67×10²</td>
<td>8.69×10²</td>
<td>8.69×10²</td>
</tr>
<tr>
<td>Internal resistance</td>
<td>1.17</td>
<td>1.23</td>
<td>1.20</td>
</tr>
<tr>
<td>Capacitance (pF)</td>
<td>2.07×10⁴</td>
<td>2.03×10⁴</td>
<td>2.04×10⁴</td>
</tr>
<tr>
<td>Q</td>
<td>5.63×10⁵</td>
<td>5.36×10⁵</td>
<td>5.49×10⁵</td>
</tr>
<tr>
<td>Resonant frequency (kHz)</td>
<td>1.19×10⁶</td>
<td>1.20×10⁶</td>
<td>1.20×10⁶</td>
</tr>
</tbody>
</table>

### Table 2: Parameters and symbol characters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol character</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Self inductance</td>
<td>L₁, L₂</td>
<td>1: transmitter</td>
</tr>
<tr>
<td>Resonator capacitance</td>
<td>C₁, C₂</td>
<td>2: receiver</td>
</tr>
<tr>
<td>Internal resistance</td>
<td>R₁, R₂</td>
<td></td>
</tr>
<tr>
<td>Angular velocity</td>
<td>ω₀</td>
<td>At resonant frequency</td>
</tr>
<tr>
<td>Mutual inductance</td>
<td>L_m</td>
<td></td>
</tr>
<tr>
<td>Load resistance</td>
<td>R_L</td>
<td>For each phase</td>
</tr>
<tr>
<td>Input, output voltage</td>
<td>V₁, V₂</td>
<td>Ratio: Aᵥ=V₂/V₁</td>
</tr>
<tr>
<td>Input, output current</td>
<td>I₁, I₂</td>
<td>Ratio: Aᵢ=I₂/I₁</td>
</tr>
<tr>
<td>Coupling coefficient</td>
<td>k</td>
<td>k=Lₘ/(L₁L₂)⁰.⁵</td>
</tr>
<tr>
<td>Efficiency</td>
<td>η</td>
<td>η=Aᵥ/Aᵢ</td>
</tr>
</tbody>
</table>

### 4 Equivalent circuit calculation

#### 4.1 Equivalent circuit of single-phase WPT

Before the analysis of three-phase system, equivalent circuit of conventional single-phase system [6] should be introduced. Single-phase system with LC resonators is described as figure 10. The resonant frequency \( f \) of this circuit is calculated as equation (5).

\[
f = \frac{ω_b}{2π} \cdot \frac{1}{2π\sqrt{L_1C_1}} = \frac{1}{2π\sqrt{L_2C_2}} \tag{5}
\]

On this equivalent circuit, output voltage \( V₂ \) and output current \( I₂ \) is describable with input voltage \( V₁ \) and input current \( I₁ \) thus the ratios of these voltage and current is described as equation (6) and (7).

\[
Aᵥ = j \frac{ω_bL_mR_1}{R_1R_L + R_1R_2 + (ω_bL_m)^2} \tag{6}
\]

\[
Aᵢ = j \frac{ω_bL_m}{(R_L + R_2)} \tag{7}
\]
Figure 4: Coupling coefficient (measured)

Figure 5: Voltage ratio

Figure 6: Current ratio

Figure 7: Power ratio (Transfer efficiency)

Figure 8: Phase of electric angle

Figure 9: Optimal resistance
The power factor of this circuit is always 1 in resonant frequency. Transfer efficiency \( \eta \), therefore, is described as equation (8)
\[
\eta = A \cdot \bar{A} = \frac{\omega_0 L_m R_L}{(R_1 + R_2)(R_t R_L + R_t R_2 + (\omega_0 L_m)^2)}
\]
These equations from (6) to (8) are the basic analysis formulas of magnetic resonant coupling. Resonant frequency and resistance values in these formulas are setting values, which are already known. Therefore these ratios can be calculated if mutual inductance \( L_m \) is measured in experiment.

4.2 Modification of the equations for three-phase analysis

The equivalent circuit in previous subsection is just for single-phase system. The next step is modification of these formulas into three-phase system. Difference between single-phase and three-phase is number of mutual inductance values. Single-phase system has one resonator for transmitter and one resonator for receiver, so mutual inductance \( L_m \) is unique. Three-phase system, in contrast, has three resonators for transmitter and three resonators for receiver, so mutual inductance values between transmitter and receiver are nine values. Moreover, cross coupling [7] of mutual inductance exists between next resonators in transmitter or receiver. In this paper, the internal cross coupling values are ignored because cross coupling value is much smaller than mutual inductance between transmitter and receiver. Total number of mutual inductance value is nine in this system, which means three times three, thus three values are concerned with each resonator. For example, phase a resonator in receiver side has mutual inductance with resonators of phase a, b, and c in transmitter side, which are described as \( L_{\text{max}} \), \( L_{\text{mca}} \), \( L_{\text{mba}} \). These three values should be composited into one value, and then the value can be applied to the analysis formulas (6) to (8). The composite value is a kind of effective value. Composition method is liner combination of trigonometric functions. Each phase of transmit resonators has 120° phase difference, therefore the composition is described as equation (9) and (10).
\[
L_{\text{max}} \sin(\alpha x) + L_{\text{mca}} \sin(\alpha x + \frac{\pi}{3}) + L_{\text{mba}} \sin(\alpha x + \frac{2\pi}{3}) = L_{\text{max}} \sin(\alpha x + \phi)
\]
(9)
\[
L_{\text{valo}} = \sqrt{L_{\text{max}}^2 + L_{\text{mca}}^2 + L_{\text{mba}}^2 - (L_{\text{max}} L_{\text{mca}} + L_{\text{mca}} L_{\text{mba}} + L_{\text{mba}} L_{\text{max}})}
\]
(10)
The phase shift between transmitter and receiver is described as equation (11). Angle datum of the phase is the phase of resonator in transmitter.
\[
\sin \phi = \frac{(\sqrt{3}/2)L_{\text{valo}} - (\sqrt{3}/2)L_{\text{max}}}{L_{\text{max}} - (L_{\text{mca}}/2)^2 - (L_{\text{mba}}/2)^2}
\]
\[
\cos \phi = \frac{L_{\text{valo}}}{L_{\text{max}} - (L_{\text{mca}}/2)^2 - (L_{\text{mba}}/2)^2}
\]
(11)

4.3 Theoretical value calculation for the experimental result

The experimental results of ratios are replicated by equation (6) to (11) and measured mutual inductance. The mutual inductance is shown in figure (4) as coupling coefficient \( k \). Transform formula between \( k \) and \( L_m \) is equation (1). The load resistance \( R_L \) is 50 \( \Omega \) and other parameters are from table 1. The results of calculation are shown on figure 5 to 7. Figure 8 shows phase as electric angle calculated from equation (11).

4.4 Optimal load resistance value for maximum transfer efficiency

In the experiment, the load resistance \( R_L \) is fixed to 50 \( \Omega \). Equation (8) shows transfer efficiency depends on mutual inductance \( L_m \) and load resistance \( R_L \). \( L_m \) depends on the position of resonators as figure 4. Variable \( R_L \) may maximize efficiency in every rotation angle. Optimal value of \( R_L \) is calculated as following equations (12) to (15). Function \( f(R_L) \) is defined in equation (12), when \( L_m \) is a fixed parameter.
\[
\frac{\eta}{\omega_0 L_m} = \frac{R_L}{R_L + \frac{1}{2} \left[ R_L R_1 + (\omega_0 L_m)^2 \right] R_L + \frac{1}{2} \left[ R_L R_1 + (\omega_0 L_m)^2 \right] R_1}
\]
(12)
Optimal \( R_L \) minimizes the value of function \( f(R_L) \). If minimum value of \( f(R_L) \) exists, differential of \( f(R_L) \) is 0 when \( R_L \) is optimal value.
\[
\frac{d}{dR_L} \left( \frac{\eta}{\omega_0 L_m} \right) = f(R_L) - R_L \left[ f'(R_L) \right] = \frac{g(R_L)}{f(R_L)^2}
\]
\[
g(R_L) = -R_L R_1^2 + \frac{1}{2} \left[ R_L R_1 + (\omega_0 L_m)^2 \right] R_1
\]
(13)
\( \{f(R_L)\}^2 \) is always positive, thus function \( g(R_L) \) rules equation (13). Then optimal \( R_L \) is calculated as equation (15).

\[
g(R_L) = 0 \iff R_L = \sqrt{R_2^2 + \frac{R_1}{R_i}(\omega_0 L_m)^2} \quad (15)
\]

Equation (15) shows optimal \( R_L \) value depends on internal resistance and mutual inductance. Therefore optimal \( R_L \) depends on the position of resonators, i.e. rotation angle rules \( R_L \) value. Optimal \( R_L \) calculated for each rotation angle as shown in figure 9.

5 Discussion

5.1 Experiment analysis

Results of the experiment show that three-phase WPT is possible. Efficiency of experimental result is around 80\%, expect in 60 ± 120°. Efficiency in rotation angle 60 ± 120° is lower than the other angles because there is not sufficient mutual inductance shown in figure 4. Theoretical calculation provides accurate ratio in almost all rotation angle compared to experimental results. Therefore equivalent circuit is possible to apply on three-phase system analysis and linear combination of trigonometric function is suitable for transform method from three-phase to single-phase formulas. The error between measured value and theoretical value is less than 10\% except in rotation angles of 60 ± 120°, which are angles of lower transfer efficiency. The reason of larger error in these angles is caused by smaller mutual inductance and ignorance of cross coupling. Measurement error may always exist but the effect of the error is deferent in each angle. Where measured value is large, the error value has relatively small effect on the result, and where value measured is small, the error has large effect. In these angles, mutual inductance is very small, the error effect is larger. Ignorance of cross coupling effects in the same matter, i.e. even though cross coupling value of mutual inductance is small, cross coupling has larger effect in those angle.

5.2 Optimal load resistance analysis

Calculated efficiency with optimal load suggests that three-phase system has high potential up to 95% transfer efficiency. Even in 60 ± 120°, maximum efficiency is estimated as 82\%, thus three-phase system is able to transfer in all angle of rotation. Figure 9 shows optimal load in each angle has similar shape of mutual inductance. The reason of that shape is proven from equation (15). Because internal resistance \( R_2 \) is much smaller than \( \omega_0 L_m \) as table 1, equation (15) can be transformed into equation (16).

\[
R_L = \sqrt{R_2^2 + \frac{R_2}{R_i}(\omega_0 L_m)^2} = \sqrt{\frac{R_2^2 + (\omega_0 L_m)^2}{R_i}} \approx \frac{\omega_0 L_m}{R_L} \quad (16)
\]

Figure 5 and 6 shows that the voltage ratio and the current ratio shape a circle with radius of 1. This is also proven by equation (16). Equation (6) is transformed into equation (17), and equation (7) into (18).

\[
A_v = j \frac{\omega_0 L_m R_i}{R_1 + R_2 + (\omega_0 L_m)^2} \quad (17)
\]

\[
A_i = j \frac{\omega_0 L_m}{R_1 + R_2} \approx j \frac{R_i}{R_1 + R_2} \frac{\omega_0 L_m}{R_L} \quad (18)
\]

Internal resistance \( R_1 \) and \( R_2 \) is much smaller than load \( R_L \), approximation of equation (19) consists.

\[
\frac{R_1 R_2}{R_L} \approx 0 \quad (19)
\]

Therefore equation (20) and (21) are described.

\[
A_v \approx j \frac{\omega_0 L_m}{R_1 + (\omega_0 L_m)^2} \approx j \frac{\omega_0 L_m}{R_i + (\omega_0 L_m)^2} \quad (20)
\]

\[
A_i \approx j \frac{\omega_0 L_m}{\omega_0 L_m + R_2} \approx j \frac{1}{\omega_0 L_m + (\omega_0 L_m)^2} \quad (21)
\]

6 Conclusion and future work

This paper provides a basic analysis of three-phase WPT through the experiment. The experiment proves that three-phase WPT is useful for rotating equipment. Comparison of experimental results and theoretical values proves equivalent circuit calculation method is also suitable for three-phase system. In the future, the authors are planning high power experiment to prove the advantage of three-phase WPT via magnetic resonant coupling.
Acknowledgments
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References


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